# Emergent Gravitational Response from Structural Constraints:
# A Toy Model with Observational Signatures in Galaxy Rotation Curves

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## Abstract

We propose a phenomenological framework in which the effective gravitational response emerges from structural constraints on accessible information. We show that the inferred compressibility profiles organize into a small number of robust dynamical classes and admit a simple description when expressed in terms of an accumulated structural variable.

Using SPARC rotation curve data, we reconstruct $\kappa(r)$ profiles and show that they do not follow a universal functional form. Instead, they organize into a small number of distinct morphological classes. We then introduce an accumulated structural variable,

$$S(r) = \int_0^r \left| \frac{d \log V_{\text{bar}}}{dr'} \right| dr',$$

and show that the resulting $\kappa(S)$ relations are significantly more regular and admit simple class-dependent logistic descriptions.

A clean-transition class is associated with a delayed and relatively sharp transition in $S$, a complex class with a broader transition, and a strong-discrepancy class with an effectively early transition. These patterns remain qualitatively robust under moderate variations of the stellar mass-to-light ratios and under alternative definitions of the structural density used to construct $S$.

The results suggest that the observed mass discrepancy may reflect organized dynamical regimes governed by an accumulated structural variable rather than a single universal correction law.

## 1 Introduction

The nature of gravity at galactic and cluster scales remains an open problem. Observations such as galaxy rotation curves [4] and cluster collisions [1] are typically interpreted within the ΛCDM paradigm [2].

Alternative approaches such as MOND [5] modify gravitational laws. In this work, we explore a different route: a phenomenological framework in which the effective gravitational response depends on structural organization and information accessibility.

The present approach is not proposed as a fundamental replacement for existing paradigms. Rather, it is intended as an empirical and phenomenological model capable of identifying structured signatures in observational data and of motivating more detailed theoretical development.

## 2 Conceptual Framework

We define an informational density

$$\mathcal{I}(x) = \rho_{\mathrm{bary}}(x)\chi(x),$$

where $\chi(x)$ encodes structural organization, such as density gradients, local compactness, or morphological transitions.

We define a compressibility field

$$\kappa(x) \in [0, 1],$$

which may be interpreted as an effective order parameter encoding the degree of local information accessibility.

The effective source is

$$R(x) = (1 - \kappa(x))\mathcal{I}(x).$$

The gravitational potential satisfies the effective closure

$$\nabla^2\Phi = 4\pi G\rho_{\mathrm{bary}}(1 + \xi(1 - \kappa)\chi).$$

This relation should be interpreted as a phenomenological closure rather than a fundamental field equation. This relation is not derived from first principles, but introduced as a phenomenological closure consistent with the interpretation of $\kappa$ as an effective order parameter. The central idea is that dynamical discrepancy may be linked not only to the amount of baryonic matter, but also to how that matter is structurally organized.

# 3 Toy Model and Observational Proxy

At the phenomenological level, an operational proxy for the compressibility profile is

$$\kappa(r) = \frac{V_{\mathrm{bar}}^2(r)}{V_{\mathrm{obs}}^2(r)},$$

where

$$V_{\mathrm{bar}}^2(r) = V_{\mathrm{gas}}^2(r) + \Upsilon_d V_{\mathrm{disk}}^2(r) + \Upsilon_b V_{\mathrm{bul}}^2(r).$$

This immediately implies

$$V_{\mathrm{obs}}^2(r) = \frac{V_{\mathrm{bar}}^2(r)}{\kappa(r)},$$

so that departures of $\kappa$ from unity encode the observed dynamical discrepancy.

When $\kappa(r) \approx 1$, the observed dynamics is close to baryonic expectations. When $\kappa(r) < 1$, the observed velocity exceeds the baryonic prediction.

# 4 SPARC-Based Reconstruction

Using the SPARC dataset [3], we reconstruct $\kappa(r)$ for a large sample of galaxies and study the radial structure of the resulting profiles.

A first empirical finding is that the most significant changes in $\kappa(r)$ often occur close to structural transitions in the baryonic distribution. This motivates the interpretation that the relevant observable may not be a universal correction law, but the presence of regime transitions in the inferred compressibility profile.

We also tested several simple functional hypotheses for $\kappa(r)$:

- single sigmoid transitions in radius,
- local structural models of the form $\kappa = \kappa(S)$,
- non-local structural models based on kernelized proxies.

These simple closed-form models generally failed to capture the observed diversity of profiles across the SPARC sample, indicating that $\kappa(r)$ does not admit a simple universal functional description. This suggests that $\kappa(r)$ is not well described by a single universal function of radius or of basic structural proxies.

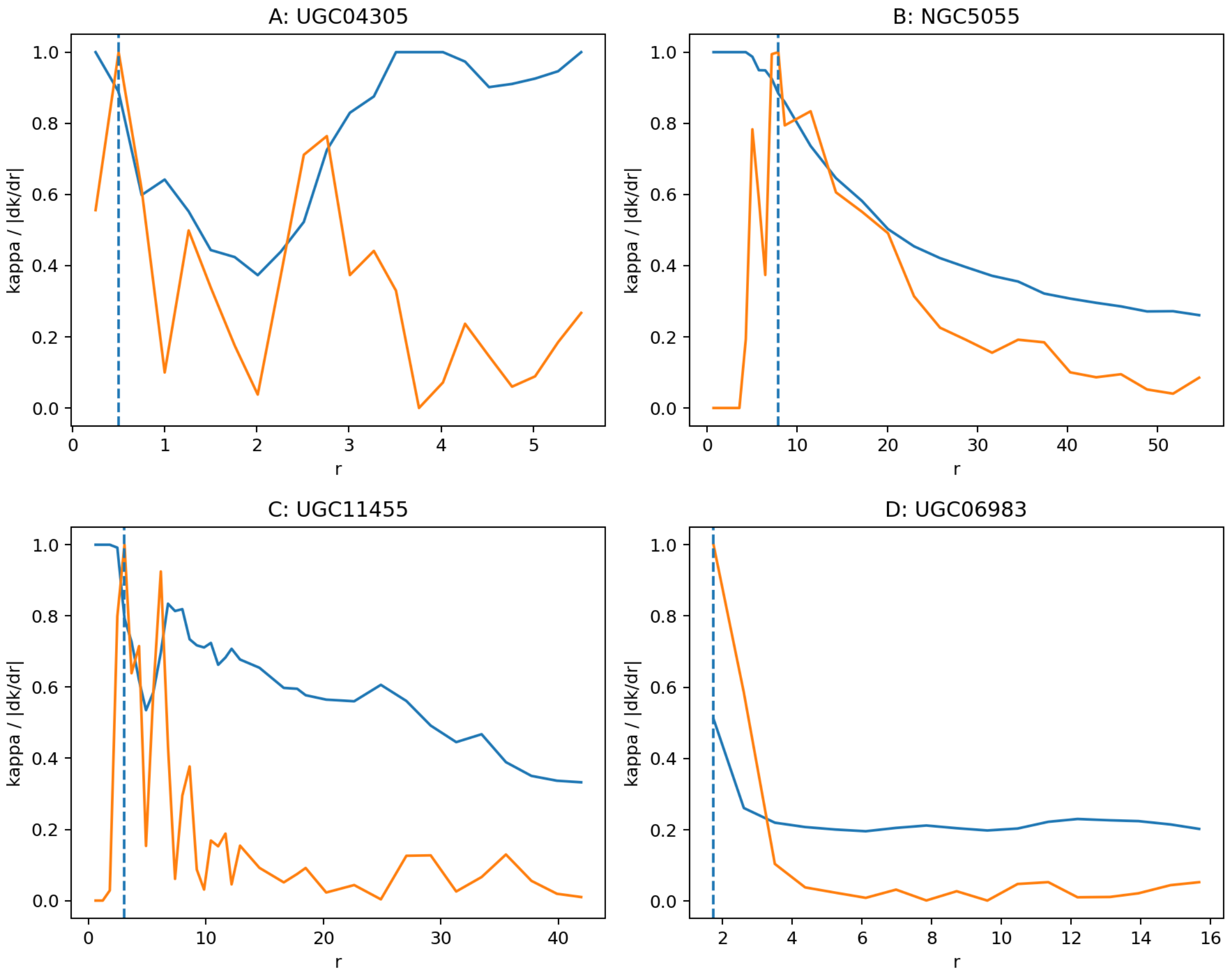

Figure 1: Representative examples of inferred compressibility-profile classes. The blue curves show $\kappa(r)$, while the orange curves show the normalized gradient $|d\kappa/dr|$ (rescaled for visualization purposes). Dashed vertical lines indicate dominant transition radii.

# 5 Morphology of Compressibility Profiles

The inferred compressibility profiles $\kappa(r)$ do not follow a single universal functional form across the SPARC sample. Instead, they organize into a small number of distinct morphological classes.

Using the baseline baryonic modeling adopted here, the sample is dominated by a strong-discrepancy class (74 galaxies), followed by a complex class (34), a clean-transition class (15), and a residual quasi-baryonic class (1).

Representative examples are shown in Fig. 1. The clean-transition class is especially notable because it shows an inner regime with $\kappa \approx 1$ followed by a relatively sharp outer decline. The complex class exhibits a broader and more irregular decline, while the strong-discrepancy class remains at low $\kappa$ over most radii.

These classes show clear correlations with baryonic structure, including gas dominance in outer regions and inner baryonic concentration. This indicates that the radial structure of the discrepancy is organized rather than stochastic. Clean-transition systems tend to exhibit a strongly baryon-dominated inner region, while complex profiles are associated with systems displaying more intricate structural features. Strong-discrepancy systems show a higher gas fraction in outer regions.

## 6 Structural Reparametrization via an Accumulated Variable

The empirical class structure suggests that radius itself may not be the most natural variable. We therefore introduce an accumulated structural quantity

$$S(r) = \int_0^r \sigma(r')\, dr',$$

with the minimal phenomenological choice

$$\sigma(r) = \left| \frac{d \log V_{\mathrm{bar}}}{dr} \right|.$$

This variable measures the cumulative structural variation of the baryonic profile. In discrete form, for SPARC-like radial sampling, $S(r)$ is obtained through cumulative integration of $\sigma(r)$ along the observed rotation-curve points.

The motivation for this reparametrization is that the system may respond not to radius alone, but to the accumulated structural complexity encountered up to that radius.

## 7 Class-Dependent $\kappa(S)$ Relations

When expressed in terms of $S$, the inferred compressibility profiles become substantially more regular. This represents a significant simplification relative to $\kappa(r)$, where no comparable near-universal description was found. In particular, the profiles are well described by class-dependent logistic relations of the form

$$\kappa_C(S) = \kappa_{\infty,C} + \frac{1 - \kappa_{\infty,C}}{1 + \exp\left(\frac{S - S_{c,C}}{\Delta_{S,C}}\right)},$$

where $C$ denotes the profile class, $S_{c,C}$ is a transition scale, $\Delta_{S,C}$ the transition width, and $\kappa_{\infty,C}$ the asymptotic low-$\kappa$ value.

A pooled fit across 119 galaxies showed that $\kappa(S)$ is generally much more regular than $\kappa(r)$ and admits low logistic fit errors. More importantly, separate class fits reveal distinct families rather than a single universal law.

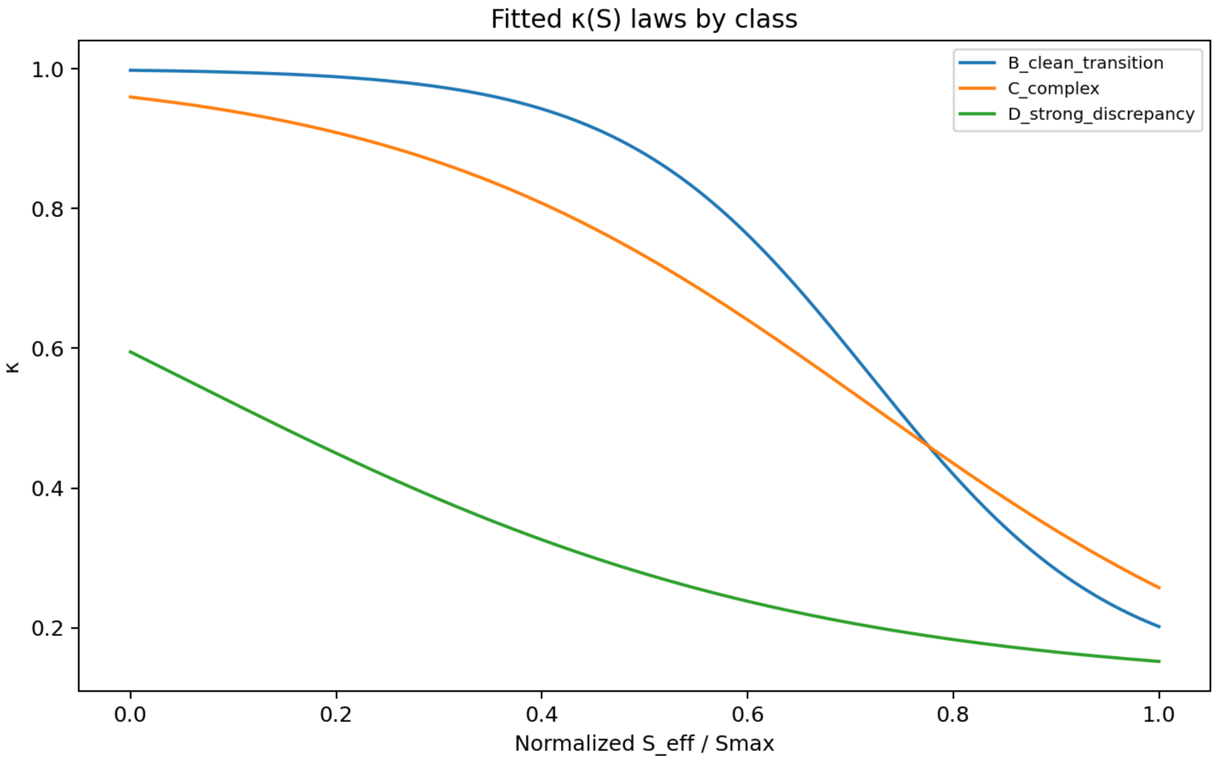

Figure 2: Class-dependent fitted $\kappa(S)$ laws obtained from the pooled SPARC sample. The clean-transition, complex, and strong-discrepancy classes occupy distinct logistic families rather than collapsing to a single universal curve.

The fitted laws for the three main classes are shown in Fig. 2, which represents one of the central empirical results of this work, and representative parameter values are summarized in Table 1.

Several features are noteworthy. First, the clean-transition and complex classes are not distinguished primarily by the location of the transition scale, but by the width of the transition. Second, the strong-discrepancy class is characterized by a much smaller effective $S_c$, suggesting that such systems enter the discrepant regime at very low accumulated structural scale.

## 8 Robustness Tests

The qualitative class structure of $\kappa(r)$ appears robust under moderate variations of the adopted stellar mass-to-light ratios. Varying $\Upsilon_{\rm disk}$ from 0.4 to 0.6 and $\Upsilon_{\rm bulge}$ from 0.6 to 0.8 changes the assigned class for only about 7–8% of the galaxies in the tested sample.

The $\kappa(S)$ class structure is also qualitatively robust under these same variations. In particular, the clean-transition class retains a very stable fitted transition scale under the tested baryonic modeling changes, while the complex class remains broader but still transitions in a comparable range.

| Class | $N_{\text{gal}}$ | $\kappa_\infty$ | $S_c$ | $\Delta_S$ |
|---|---|---|---|---|
| Clean transition (B) | 15 | 0.125 | 0.719 | 0.12 |
| Complex (C) | 34 | 0.039 | 0.719 | 0.23 |
| Strong discrepancy (D) | 69 | 0.116 | 0.050 | 0.30 |

Table 1: Representative logistic parameters for the three main $\kappa(S)$ families. The clean-transition and complex classes share a similar transition scale but differ strongly in transition width, while the strong-discrepancy class exhibits an effectively early transition.

We also tested alternative definitions of the structural density $\sigma(r)$, replacing $d \log V_{\text{bar}}/dr$ with variants based on $V_{\text{bar}}^2$ and $g_{\text{bar}}$. The numerical value of $S_c$ does depend on this choice, indicating that $S$ is not yet a canonical variable. However, the existence of class-dependent logistic $\kappa(S)$ relations is preserved. Thus, what appears robust is not a single exact numerical value of $S_c$, but the existence of an accumulated structural variable organizing the response.

## 9 Bullet-Cluster-like Toy Models

Cluster collisions provide a critical test for any framework that aims to explain gravitational discrepancy without introducing an explicit dark matter component.

This section is exploratory and intended only as a qualitative consistency check. We consider a toy picture in which gas becomes diffuse while compact structures remain localized. The mechanism is:

1. gas spreads and is spatially diluted,
2. compact structures remain coherent,
3. the compact sector is structurally amplified,
4. the gas contribution is diluted by broad mediation.

This leads to the qualitative condition

$$\frac{J_{\text{off}}}{J_{\text{center}}} > 1,$$

that is, off-center peaks in the effective response can dominate over the central gaseous contribution.

We do not claim quantitative agreement with observed lensing maps; this remains a qualitative proof-of-principle rather than a quantitative model.

# 10 Interpretation of $S$, $S_c$, and $\Delta_S$

The accumulated variable $S$ should not be interpreted merely as a convenient reparametrization. Within the present framework, it can be read as an effective accumulated structural-information measure: a quantity that grows as the baryonic profile accumulates local structural variation.

On this reading, the transition scale $S_c$ is not simply a fitted constant but the accumulated structural scale at which the near-baryonic response ceases to be sustained. Below this threshold, the system remains in a regime with $\kappa \approx 1$ or only mild discrepancy. Once $S$ exceeds $S_c$, the effective response reorganizes toward a lower-$\kappa$ regime.

The width $\Delta_S$ then measures how sharply this reorganization occurs. The distinction between the clean-transition and complex classes is therefore naturally interpreted not as the absence or presence of a transition, but as the sharpness with which the transition takes place.

This interpretation is deliberately modest. We do not claim that $S$ has yet been derived from a microscopic theory, nor that $S_c$ is universal in a strict sense. Rather, the results are consistent with the hypothesis that galactic dynamics may be governed by transitions controlled by an accumulated structural quantity, with class-dependent transition laws.

# 11 Discussion

A key observational result is that the inferred compressibility profiles do not appear random. Instead, they organize into a small number of morphological classes with plausible physical interpretation.

This is significant because the previous tests indicate that simple universal laws for $\kappa(r)$ fail, whereas a classification-based view reveals structured behaviour. The accumulated variable $S$ strengthens this conclusion: it does not erase the class structure, but reorganizes it into a small family of logistic response laws.

This suggests that classification-based approaches may provide a more robust empirical entry point than direct functional modelling. The data do not favour a single universal $\kappa(S)$ law; they favour a small number of class-dependent laws.

The search for a simple proxy determining $S_c$ from a few structural observables was inconclusive. This is consistent with the view that $S_c$ may be an emergent regime parameter rather than a trivial function of a single local observable. If so, its eventual physical interpretation may require a more genuinely non-local or informational description.

# 12 Conclusions

We have proposed a phenomenological framework in which the effective gravitational response is encoded in an inferred compressibility profile $\kappa(r)$.

Analysis of SPARC rotation curves shows that $\kappa(r)$ profiles do not follow a universal law, but instead organize into a small number of morphological classes. Introducing the accumulated structural variable

$$S(r) = \int_0^r \left| \frac{d \log V_{\mathrm{bar}}}{dr'} \right| dr'$$

reveals a more regular organization:

$$\kappa = \kappa_C(S),$$

with distinct logistic laws for the main classes.

These class-dependent relations appear qualitatively robust under moderate changes in the stellar mass-to-light ratios and under alternative definitions of the structural density used to build $S$. The results therefore support the view that the observed dynamical discrepancy may reflect organized regime transitions rather than a single universal modification law.

**Key observational result.** The existence of distinct $\kappa(S)$ families indicates that galaxy dynamics may be governed by class-dependent transitions controlled by an accumulated structural variable.

If confirmed, this would point toward a description of galactic dynamics in which structural organization plays a direct and previously unrecognized role in shaping the effective gravitational response.

# References

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